Response of Structures to Random Acoustic Excitation—An Experimental Study

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Nomenclature

a, b = sides of rectangular plate
d = space correlation
h = thickness of plate
m, n, r, s = positive integers
p = pressure parameter
p_o = actual pressure
t = time parameter
t_o = actual time
w = transverse deflection of plate
x, y = coordinates of points in the middle surface of plate
D = bending stiffness of plate
E = modulus of elasticity of plate material
H_{mn} = frequency ratio
K_{mn} = dimensionless damping parameter
R = autocorrelation function
T = time duration of the random process
V_{mn}, V_{rs} = eigenfunctions of plate
β = damping parameter
β_o = actual damping coefficient
ξ, η = coordinates of points in the middle surface of plate
$\lambda_{mn}, \lambda_{rs}$ = eigenvalues of plate
τ = time delay
ω = circular frequency parameter
ω_o = actual circular frequency
γ = Poisson's ratio
ρ = mass density of plate material
Φ = power spectral density
ζ = damping ratio

Theme

EXPERIMENTAL investigation of the response of structures to a peaked type of power spectral density of loading, which represents the actual random excitations realistically, is discussed. Response accelerations have been measured for structures like flat and stiffened plates and have been analyzed for their power spectral density and autocorrelation functions. The random loading is uniform over the plate area so that the space correlation may be assumed unity. Experimental findings in this paper are compared with theoretical results and the agreement between the two is found to be quite good.

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Introduction: The response of launch and space vehicles to the random acoustical environment around it is of great interest from the point of its influence on internal components and systems in addition to that of structural integrity itself. The factors influencing the acoustic excitation process and the subsequent response are numerous and still not clearly understood. Hence, theoretical analyses based on some idealized mathematical models will give only a broad nature of the phenomena. It is of utmost importance that extensive environmental testing should be carried out to get an insight into the response behavior of structure subjected to such random excitations.

Analytical considerations: The response of flat and stiffened plates to a peaked type of power spectral density of loading is analyzed in Refs. 1–3. The form of the power spectral density of loading assumed in normalized form, is

$$\Phi_N(\omega) = \frac{8\pi^{1/2}}{\omega^*}\left(\frac{\omega}{\omega^*}\right)^2 \exp\left[-(\omega/\omega^*)^2\right] \tag{1}$$

where ω^* is the frequency at which the curve is peaked. The power spectral density of acceleration and stress as well as the corresponding cross correlations were derived using Eq. (1), using generalized harmonic analysis and normal mode approach for the cases of flat plates with simply supported and built-in edge conditions.

Since the loading is peaked, taking only a single term corresponding to the natural frequency near the peak of the loading power spectral density, the cross correlations of acceleration and stress for a built-in plate are given, respectively, by

$$R_{\ddot{\omega}}(x, y, \xi, \eta, \tau) = \frac{0.842\overline{p^2}(1+K_{mn}^2)}{H_{mn}^3 K_{mn}} V_{mn}(x, y) V_{mn}(\xi, \eta) \times \exp\left[\frac{1-K_{mn}^2}{H_{mn}^2(1+K_{mn}^2)} - \beta\tau\right] \times \cos\left[K_{mn}\beta\tau - \frac{2K_{mn}}{H_{mn}^2(1+K_{mn}^2)} + 5\theta_{mn}\right]\cdots \tag{2}$$

and

$$R_{\sigma_x}(x, y, \xi, \eta, \tau) = \frac{30.4\overline{p^2}D^2(1+K_{mn}^2)}{h^4 a^4 H_{mn}^3 K_{mn} \lambda_{mn}^8} \times \left[\frac{\partial^2 V_{mn}(x, y)}{\partial x^2} + \gamma \frac{\partial^2 V_{mn}(x, y)}{\partial y^2}\right] \times \left[\frac{\partial^2 V_{mn}(\xi, \eta)}{\partial \xi^2} + \gamma \frac{\partial^2 V_{mn}(\xi, \eta)}{\partial \eta^2}\right] \exp\left[\frac{1-K_{mn}^2}{H_{mn}^2(1+K_{mn}^2)} - \beta\tau\right] \times \cos\left[K_{mn}\beta\tau - \frac{2K_{mn}}{H_{mn}^2(1+K_{mn}^2)} + \theta_{mn}\right]\cdots \tag{3}$$

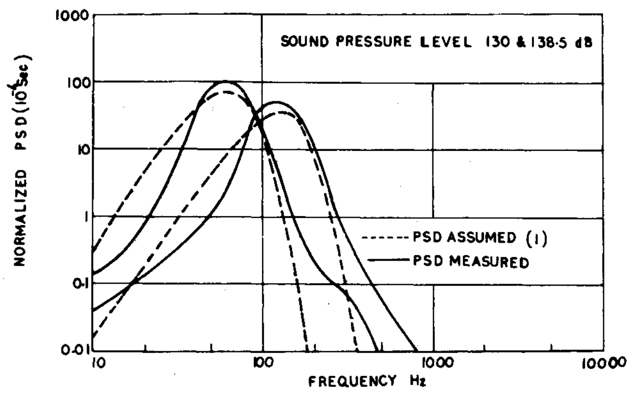

Fig. 1 Measured normalized power spectral density of acoustic excitation with peak frequency of 63 and 125 Hz.

where

$$H_{mn} = \omega^*/\lambda_{mn}^2 \qquad K_{mn} = [(\lambda_{mn}^4/\beta^2) - 1]^{1/2}$$
$$\theta_{mn} = \tan^{-1}(1/K_{mn}) \tag{4}$$

In Eqs. (2) and (3) the time, damping, and pressure parameters, t, β, and p, respectively, are related to the actual values of time, damping and pressure t_o, β_o, and p_o, respectively, by the following relations:

$$t = t_o(D/\rho h)^{1/2}, \qquad \beta = \tfrac{1}{2}\beta_o(1/\rho h D)^{1/2}$$
$$p = p_o/D, \qquad D = Eh^3/12(1-\gamma^2) \tag{5}$$

The frequency parameter ω is related to the actual circular frequency ω_o by

$$\omega = \omega_o(\rho h/D)^{1/2}$$

and $V_{mn}(x, y)$ is the mode function of the built-in plate. The mean square values of acceleration and stress at the center of the plate are obtained by putting $x = \xi = a/2$, $y = \eta = b/2$, and $\tau = 0$ in Eqs. (2) and (3).

The response of stiffened plates for a peaked power spectral density of loading is described in Ref. 3 using an approximate method proposed by Lin.[4]

Experimental investigations: The experimental set-up consists of an exponential horn driven by an electropneumatic transducer. The transducer consists of an electrodynamically operated pneumatic valve which responds to any type of applied electrical wave form and controls or modulates the velocity of an airstream supplied from a compressor.

The instrumentation for the measurement and analysis of noise consisted of noise generator, condenser microphones, piezoelectric accelerometer, tape recorder, frequency analyzer, band-pass filter, level recorder, autocorrelator, etc. The structures tested were: 1) a flat plate with built-in edges; 2) a flat plate with simply-supported edges; and 3) a skin-stringered panel. However, the case of the flat plate with built-in edges only is discussed here.

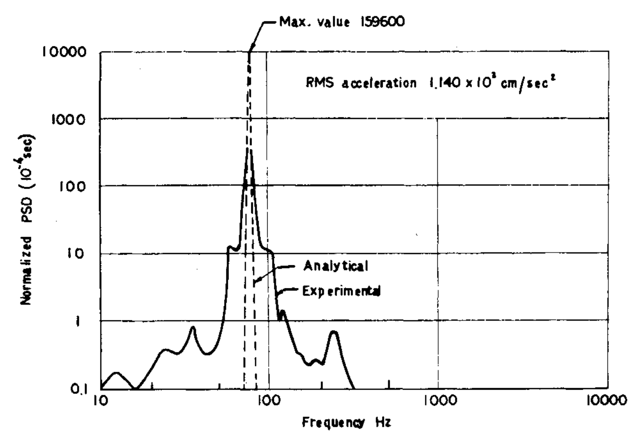

Fig. 2 Measured normalized power spectral density of acceleration response at the center of built-in plate ($f^* = 63$ Hz).

Table 1 Comparison of theoretical and experimental results

Mode no. "mn"	Natural frequency in Hz		f^a	SPL in db	Mean square acceleration response $\bar{\omega}^2/p^2$	
	Experimental	Theoretical	in Hz		Experimental	Theoretical
"11"	78	79	63	130.0	94.0	99.2
"12"	156	161	...	...	...	...
"22"	232	237	...	...	...	...
"13"	275	289	250	138.5	60.9	54.7
"23"	363	362	...	...	...	...
"14"	...	460	...	...	...	...
"33"	495	482	500	137.0	26.0	17.8
"24"	560	531	...	...	...	...

$^a f$—Center frequency of narrow band excitation. Measured damping ratio for the fundamental mode is $\zeta_{11} = 0.02$, $K_{11} = [(1-\zeta_{11}^2)^{1/2}]/\zeta_{11} = 50.0$.

Natural frequencies were obtained both theoretically and experimentally (by subjecting the structure to sinusoidal excitation), for the built-in plate. The damping parameter for the fundamental mode, i.e., K_{11}, was obtained by a free vibration test, which can be used in computing the damping parameters for the higher modes.

A narrow band random loading was obtained by passing a white noise signal through octave band pass filters and fed to the electropneumatic transducer through an amplifier. A narrow band random loading was used, since it was not possible to construct a loading according to Eq. (1). The normalized, measured power spectral densities of the acoustic excitation are shown in Fig. 1 along with the analytical substitutes as per Eq. (1), and the agreement is quite good.

The acceleration response of the structure was picked up by the accelerometer and recorded in a tape recorder along with the excitation noise picked up with a microphone simultaneously. The input acoustic excitation and the acceleration response signals from the tape recorder were separately analyzed using the frequency analyzer, $\frac{1}{3}$ octave filter, and the level recorder. The autocorrelation of the acceleration response was measured using the analog correlator along with the time delay unit.

Table 1 presents the experimental and theoretical values of the natural frequencies of the structure, along with the sound pressure levels of the noise loading and the nondimensional acceleration response. Only the modes "1," "13," and "33" whose natural frequencies (experimental) are 78, 275, and 495 Hz, respectively, are excited by the acoustic excitation. The experimental and analytical response values agree well.

The normalized, measured power spectral density of the acceleration response of the built-in plate is shown in Fig. 2. The difference between the theoretical and experimental curves is due to the fact that the $\frac{1}{3}$ octave bond was not narrow enough to measure the extremely narrow band response of the structure. However, a good qualitative agreement exists between the two results.

Normalized autocorrelation function of the acceleration response of the built-in plate was also measured. Since the response is a very narrow band one, the autocorrelation curve resembles that of a harmonic phenomenon but decreasing in amplitude with time.

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